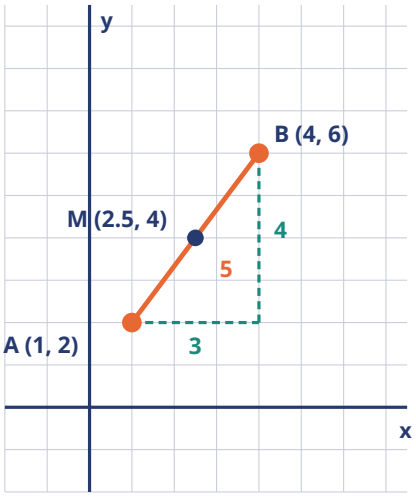
The Coordinate Plane, Distance and Midpoint

Every point is written as **(x, y)**: x says how far across, y says how far up or down.

Quadrant	Where	Sign of (x, y)	Example
I	top right	(+, +)	(3, 2)
II	top left	(−, +)	(−3, 2)
III	bottom left	(−, −)	(−3, −2)
IV	bottom right	(+, −)	(3, −2)

On the x-axis, y = 0. On the y-axis, x = 0. The origin is (0, 0).



Points A (1, 2) and B (4, 6)

Horizontal gap
4 − 1 = 3

Vertical gap
6 − 2 = 4

Distance AB
$\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$

Midpoint M
(5/2, 8/2) = (2.5, 4)

Two essential formulas

Formula	For points (x_1, y_1) and (x_2, y_2)
Distance	$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
Midpoint	$M = \left((x_1 + x_2)/2 , (y_1 + y_2)/2 \right)$

Remember

- The distance formula is Pythagoras in disguise: the horizontal and vertical gaps are the two shorter sides.
- Subtract in the same order for both x and y. Squaring removes any negative sign.
- The distance from the origin to (x, y) is $\sqrt{(x^2 + y^2)}$.

Straight Lines and More Formulas

A line is described by how steep it is (slope) and where it crosses the y-axis.

Idea	Formula	Notes
Slope	$m = (y_2 - y_1) \div (x_2 - x_1)$	rise over run
Slope-intercept form	$y = mx + c$	m = slope, c = y-intercept
Point-slope form	$y - y_1 = m(x - x_1)$	line through (x_1, y_1)
Two-point form	$y - y_1 = [(y_2 - y_1)/(x_2 - x_1)] (x - x_1)$	line through two points

Types of slope

Slope	Line looks like	Example
Positive	rises from left to right	$y = 2x + 1$
Negative	falls from left to right	$y = -x + 4$
Zero	horizontal	$y = 3$
Undefined	vertical	$x = 2$

Parallel and perpendicular lines

Lines are	Slope rule	Example
Parallel	$m_1 = m_2$	$y = 3x + 1$ and $y = 3x - 5$
Perpendicular	$m_1 \times m_2 = -1$	slopes 2 and −1/2

Section formula, centroid and area (Classes 9-10)

Formula	Rule
Section formula point dividing the segment in the ratio m : n	$\left((m x_2 + n x_1)/(m + n) , (m y_2 + n y_1)/(m + n) \right)$
Centroid of a triangle	$\left((x_1 + x_2 + x_3)/3 , (y_1 + y_2 + y_3)/3 \right)$
Area of a triangle	$\frac{1}{2} \lvert x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \rvert$

Worked Examples and Practice

Try each question yourself before reading the answer.

Example 1: distance

Q. Find the distance between (1, 2) and (4, 6).

Solution. $d = \sqrt{[(4 - 1)^2 + (6 - 2)^2]} = \sqrt{(9 + 16)} = \sqrt{25} =$ **5 units**.

Example 2: midpoint

Q. Find the midpoint of (2, 6) and (8, 2).

Solution. $((2 + 8)/2 , (6 + 2)/2) =$ **(5, 4)**.

Example 3: equation of a line

Q. Find the equation of the line through (1, 3) and (3, 7).

Solution. Slope $m = (7 - 3)/(3 - 1) = 2$. Using $y - 3 = 2(x - 1)$ gives **$y = 2x + 1$**.

Example 4: section formula

Q. Find the point that divides the segment from (1, 2) to (7, 8) in the ratio 1 : 2.

Solution. $x = (1\times7 + 2\times1)/3 = 3$ and $y = (1\times8 + 2\times2)/3 = 4$, so the point is **(3, 4)**.

Your turn

1. Find the distance between (−2, −1) and (4, 7).

2. Find the midpoint of (−2, 3) and (4, −1).

3. Find the slope of the line through (2, 1) and (6, 9).

4. Write the slope and y-intercept of $y = 3x - 2$. What is the slope of a line perpendicular to it?

5. Show that the triangle with vertices (1, 1), (4, 5) and (7, 1) is isosceles.

6. Find the centroid of the triangle with vertices (0, 0), (6, 0) and (3, 9).

Answers

- 1) 10 | 2) (1, 1) | 3) 2 | 4) 3, −2, −1/3 | 5) AB = BC = 5 | 6) (3, 3)

More practice and the full guide: thepractiseground.in/blog/coordinate-geometry-graphs-visual-guide